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The Application of Fourier Techniques and Information Theory to the Assessment of Photographic Image Quality

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In the more advanced photographic systems of the type used in satellites, it is essential that the highest possible efficiency is obtained. By using the powerful tools of Shannon's Information Theory, an over-all figure of merit may be obtained for the capacity of a film to receive and store information. This involves joint consideration of the speed, contrast-transfer function, and noise power spectrum of the film. The significance and practical evaluation of this figure of merit are discussed, and examples are given for various film–developer combinations.

The Contrast-Transfer Function and the Noise Power Spectrum

In 1946 considerable impetus was given to the subject of assessing optical systems by Duffieux,[1] when he proposed the application of the same Fourier techniques which had already proved of great value in communication theory. Further developments along these lines were made by Elias, Grey, and Robinson,[2] Schade,[3] Elias,[4] Fellgett,[5] Fellgett and Linfoot,[6] and Jones.[7]

The use of Fourier theory enables image assessment to be made in terms of the Fourier spectral elements of the object and image, and by so working in the spatial frequency (u,v)-plane instead of the distance (x,y)-plane, considerable simplification can be made in the description of the performance of optical and photographic systems. Because of the close relation between spatial frequency and fineness of detail there is little loss in intuitive clarity, and today the contrast-transfer function and noise power spectrum are widely regarded as providing the most convenient means of assessing film spread and granularity.

In recent years measurements of both these functions have been made for many of the films which are available on the market.[8-14]

Information Theory

Although manufacturers of film provide a speed rating for each film, usually no rating of image quality is given. While the speed rating is usually a satisfactory guide for the ordinary photographer, it is insufficient when choosing a film for scientific purposes where the greatest possible amount of information has to be recorded by the film. Here it may prove more than worthwhile if the efficiency can be increased by even a small amount by a judicious choice of the best film–developer combination. A

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1. P. M. Duffieux, *L'integrale de Fourier et ses applications a l'optique*, Private publication, Rennes, 1946.
2. P. Elias, D. S. Grey, and D. Z. Robinson, *J. Opt. Soc. Am.*, **42:** 127 (1952).
3. O. H. Schade, *J. SMPTE*, **58:** 181 (1952).
4. P. Elias, *J. Opt. Soc. Am.*, **43:** 229 (1953).
5. P. B. Fellgett, *J. Opt. Soc. Am.*, **43:** 271 (1953).
6. P. B. Fellgett and E. H. Linfoot, *Phil. Trans. Roy. Soc. London, Ser. A*, **247:** 369 (1955).
7. R. Clark Jones, *J. Opt. Soc. Am.*, **45:** 799 (1955).
8. E. Ingelstam, E. Djurle, and B. Sjögren, *J. Opt. Soc. Am.*, **46:** 707 (1956).
9. R. L. Lamberts, *J. Opt. Soc. Am.*, **49:** 425 (1959); **51:** 982 (1961).
10. L. O. Hendeberg, *Arkiv Fysik*, **16:** 417, 457 (1960).
11. D. H. Kelly, *J. Opt. Soc. Am.*, **50:** 269 (1960); **51:** 319 (1961).
12. M. Tamura and H. Kubota, *J. Appl. Phys. (Japan)*, **26:** 92 (1957).
13. H. Ohzu and H. Kubota, *J. Appl. Phys. (Japan)*, **26:** 96 (1957).
14. H. Frieser, *Mitt. Forschungslab. Agfa Leverkusen-Muenchen*, **2:** 249 270 (1958).

slow film overdeveloped may well yield better results than a fast film developed normally, when both are used for the same purpose. However, no indication of this can be gained from knowledge of their respective speed ratings.

If the contrast-transfer function and noise power spectrum of each available film are known, as well as the speed, the choice of the most suitable film may be simplified. But, although both play an essential part in determining image quality, the extent to which each does so is not immediately obvious. Further, both may vary considerably and in a substantially different manner from film to film, and also will depend on the conditions of exposure and development.

These problems arise whenever a manufacturer attempts to improve one of his films. An improvement in some respects will often involve a loss in others, and it is difficult to decide whether or not an over-all improvement has been made. For example, a slight increase in speed may be of doubtful value if it is at the expense of a large increase in granularity. Clearly what is needed is some over-all figure of merit which can take into account both the image quality (determined by the contrast-transfer function and noise power spectrum) and the efficiency with which it is obtained (determined by the speed).

In 1948 C.E. Shannon[15] developed a mathematical technique of crucial importance in this connection, and the name Information Theory is now generally used to describe his results. Shannon showed that there is only one way to define a measure of amount of information which is consistent with basic intuitive notions. He further showed how, for certain types of communication system, the information conveyed about a given signal can be calculated from considerations of signal-to-noise ratio. A convenient unit of information is the binary digit, or "bit," which corresponds to the amount of information given by a "yes" or "no" answer when both were initially equiprobable.

The possible application of Information Theory to optical and photographic systems began to attract attention around 1953, both the work of Blanc-Lapierre[16] and Fellgett and Linfoot[6] being prominent. It was in fact Fellgett and Linfoot[6] (1955) who arrived at the first formulation of an informational figure of merit for a photographic film involving both the contrast-transfer function and the noise power spectrum. Recent developments in the practical application of this figure of merit have been made by Linfoot[17] and Jones.[18]*

Formulation

Fellgett and Linfoot showed that for a random object intensity distribution (under the constraint of a prescribed statistical-mean total spectral density) the mean information content recorded in the emulsion area B, at the mean density level D, is given by

$$I = B \iint_{-\infty}^{+\infty} \log_2 (1 + S(u,v))\, du\, dv \text{ bits/image} \quad (1)$$

where the signal-to-noise power ratio $S(u,v)$ is given by

$$S(u,v) = \frac{T^2(u,v) \cdot p(u,v)}{n(u,v)} \quad (2)$$

Here, $p(u,v)$ is the spectral power density of the fractional fluctuations in the object† intensity distribution, $n(u,v)$ is the spectral power density of the photographic noise at the mean recording level D, and $T(u,v)$ is the photographic contrast-transfer function at this level. Both $T(u,v)$ and $n(u,v)$ will be those values corresponding to the particular exposure–film–developer combination, but $p(u,v)$ is independent of film properties.

Informational assessments of photographic image quality relative to those random object distributions for which $p(u,v)$ is constant as (u,v) varies form the simplest analytical approach. In practice this corresponds to selecting a film by means of an informational rating when initially there is a complete lack of knowledge about the spatial distribution of the object. Also, to comply with the constraint on the total spectral density as previously mentioned, the information content is in the first place evaluated for a finite spatial frequency range. This is mathematically equivalent to assuming that $p(u,v) = 0$ for all (u,v) outside some circle $u^2 + v^2 < H^2$ in the spatial frequency plane, and then letting H tend to infinity. As soon as the circle includes the whole of the spatial frequency range in which $T(u,v)$ is effectively nonzero, the information content remains unchanged by a further increase in H.

Because of the isotropic properties of photographic film, measurements taken in one direction are sufficient to provide a complete description of film characteristics. Hence, it is convenient to use the line frequency ω, defined by $\omega^2 = u^2 + v^2$, and after making this substitution we obtain

$$I = 2\pi \int_0^H \log_2 \left(1 + \frac{T^2(\omega)\, p_0}{n(\omega)}\right) \omega\, d\omega \text{ bits/unit area} \quad (3)$$

where p_0 is the constant value of $p(\omega)$ within the range $0<\omega<H$.

Most methods of measurement yield the normalized contrast-transfer function defined by

$$\tau(\omega) = \frac{T(\omega)}{T_0}$$

* I am grateful to both Dr. Linfoot and Dr. Jones for allowing me to see their papers in advance of publication.

15. C. E. Shannon, *Bell System Tech. J.*, **27:** 379, 623 (1948).
16. A Blanc-Lapierre, *Symposium on Microwave Optics*, **2:** No. 46, McGill Univ., 1953.
17. E. H. Linfoot, *J. Phot. Sci.*, **9:** 188 (1961).
18. R. Clark Jones, *J. Opt. Soc. Am.*, **51:** 1159 (1961).

† For simplicity the object is regarded here as the intensity distribution arriving at the surface of the film. At this stage it will usually be modified already by such factors as the optical-transfer function of the camera system.

where T_0 is the absolute value at very low spatial frequency. T_0 may be obtained in terms of the macroscopic properties of the film, since by definition it is merely the ratio of the contrast in the image dD to the contrast in the object $\frac{dE}{E}$, and thus

$$T(\omega) = E\frac{dD}{dE}\tau(\omega)$$

Now the slope of the (H & D) characteristic curve is given by

$$\gamma = \frac{dD}{d(\log_{10}E)} = \frac{E}{\log_{10}e}\frac{dD}{dE} = \frac{E}{0.434}\frac{dD}{dE}$$

where the density D is measured in $\log_{10}$ units. This leads to the relation

$$T(\omega) = 0.434\,\gamma\,\tau(\omega)$$

The noise power $n(\omega)$ may similarly be expressed in terms of its normalized value $N(\omega)$ by the relation

$$n(\omega) = n_0\,N(\omega)$$

where n_0 is the spectral density of the noise at very low spatial frequency.

Equation (3) may now be written

$$I = 2\pi\int_0^H \log_2\left(1 + 0.189\,\frac{\gamma^2}{n_0}\,\frac{\tau^2(\omega)}{N(\omega)}\,p_0\right)\omega\,d\omega \text{ bits/unit area} \quad (4)$$

If E denotes the mean object intensity (exposure energy) which yields the mean image density D, then

$$I = \frac{2\pi}{E}\int_0^H \log_2\left(1 + 0.189\,\frac{\gamma^2}{n_0}\,\frac{\tau^2(\omega)}{N(\omega)}\,p_0\right)\omega\,d\omega \text{ bits/unit energy} \quad (5)$$

Either Eq. (4) or (5) may be used to measure the information capacity of a film. If picture quality is the main criterion, independent of speed considerations, then Eq. (4) will be used. Equation (5) assesses the over-all efficiency of the photographic process.

The Object Power Spectrum

In the final equation obtained for the signal-to-noise ratio, only p_0 is not a property of the film. That the information content increases as p_0 increases follows intuitive reasoning, since the larger the intensity fluctuations in the object, the higher the signal-to-noise ratio of the image, and the more the information that can be extracted from the image.

If Eqs. (4) and (5) are to be used to obtain a numerical value for the information content, some stipulation must be made about the *size* of p_0 (one assumption has already been made about the *shape* of the object power spectrum, i.e. that it is flat over the range $0 < \omega < H$). Recent approaches to this problem have been made by Linfoot and Jones.

Linfoot[17] considers the value of the information content as p_0 tends to zero, since for the resultant low-contrast images the information content *per unit* p_0 tends towards its maximum value. This approach is particularly useful for the comparison of different films, since the ability of a film to record low-contrast objects efficiently is of crucial importance in many applications. A further advantage is that this approach sidetracks the complications which arise due to the nonlinearity of the photographic process for high-contrast objects.

Jones[18] has evaluated the *maximum* information capacity by considering the object power spectrum which yields an image whose density fluctuations are peak-limited by $D_{\min}$ and $D_{\max}$. Difficulties are encountered due to photographic nonlinearity, and the variation of the noise power spectrum with mean density level. Also, no rigorous theory has been developed for the calculation of the information capacity of peak-limited communication channels. However, the numerical results obtained in this way have the advantage of immediate intuitive significance, and are useful in making comparisons between the photographic process and other types of communication systems such as television and image converter tubes.

An intermediate approach has been made in the present paper. In the following evaluation of film information capacities actual numerical values have been ascribed to p_0 corresponding to images of low to medium contrast. Such images are perhaps most commonly encountered in scientific application. The numerical results obtained in this way have been used to compare various film-developer combinations, and to investigate the dependence of the signal-to-noise ratio and information capacity on the object contrast.

It has been assumed that the photographic process is linear, and that the contrast-transfer function is independent of the contrast. For the range of contrasts considered here, the author has shown[19] that this assumption is fairly well justified. The further assumption has been made that the noise power measured at the mean density level may be taken as constant over the finite density range of the object contrast. Since the noise power increases with density level in an approximately linear manner, it seems reasonable to assume that the noise power at the mean level is a good average for the finite density range.

Necessary Measurements

A description follows of the experimental techniques involved in measuring the film properties which are necessary for the evaluation of the information capacity. Three films were examined, namely HPS (type RR), HP3 (type RH), and Micro-Neg Pan (type RC), samples of each film being obtained from batches manufactured during late 1961.

19. R. Shaw, *An Investigation of the Informational Properties of Photographic Emulsions*, Thesis, Cambridge Univ., 1961.

TABLE I. Development

Film	Developer	Time (min)
HPS	ID-11B	14
HP3	ID-11B	10.5
Micro-Neg Pan	ID-2(1 + 2)	5

The respective developers and development times were as indicated in Table I. All three films were dish developed at 20°C, using intermittent agitation.

The speed and contrast were obtained from standard sensitometric measurements of the macroscopic H & D curves. These curves are shown in Fig. 1, for exposure to white light and with the prescribed development. The log E scale is given in units of ergs/cm^2, by assuming a mean exposure wavelength of 5300 A.

Contrast-transfer-function measurements were made by exposing the film to a sinusoidally modulated beam of light. The camera consisted of the illuminating system of a microdensitometer. A long, narrow slit was focused on the film by a 16mm objective which produced a slit-image on the film surface about 2μ in width. Sinusoidal modulation was obtained by using a pair of polaroid screens placed in the collimated part of the beam. One of these was fixed; the other rotated at a constant angular velocity.

In order to translate the modulation of the exposure from time into distance across the surface of the film, the film was driven across the slit-image in a direction perpendicular to its length. Continual acceleration of the drive enabled the velocity of the film to be increased during exposure to give a line frequency varying from 5 lines/mm upwards.

The peak-to-peak contrast was controlled by a half-wave plate inserted between the polaroids. For each film this was orientated so that in the low spatial frequency range the peak-to-peak fluctuations in the developed image were approximately 0.5 density units. To keep constant the mean exposure level with increase of film velocity, a continuous optical wedge was driven across the collimated beam so that the beam intensity also increased in such a way that the product of intensity and exposure was held constant.

Following the exposure and subsequent development of the film, the same microdensitometer was

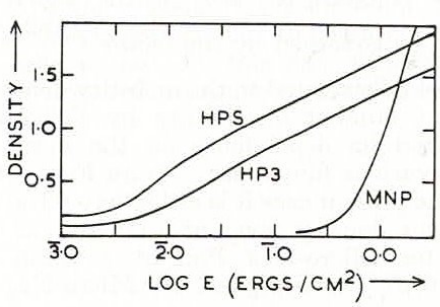

Fig. 1 Characteristic curves.

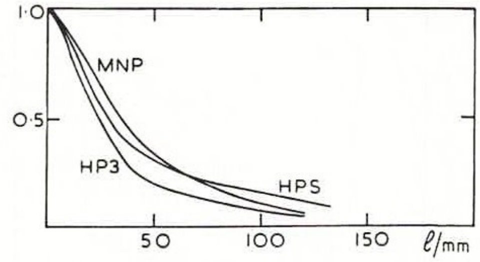

Fig. 2 Contrast-transfer functions.

used to scan the developed image. In this case the light passed through the film and onto a photomultiplier through a collecting slit of effective dimensions $0.5 \times 150\mu$ (again allowing for the magnification in imaging the film onto the slit). A nonlinear amplifier converted transmitted intensity to density, so that a direct density trace was obtained on the Honeywell recorder chart. The film contrast-transfer function was then obtained from the trace by measurement of the modulation with spatial frequency of the amplitude of the image-density fluctuations.

Strictly speaking, the contrast-transfer function so obtained includes the effects of the microdensitometer optical system and the finite width of the scanning slit, and allowance must be made for this. By the Fourier Product Theorem we have

$$\tau' = \tau \, \tau_A$$

where τ' is the measured contrast-transfer function, τ is that due to the film, and τ_A is that of the microdensitometer system. The function τ_A was measured by scanning the microdensitometer across a step input in the form of a knife-edge, and Fourier transforming the spread function obtained by numerical differentiation of the recorded edge profile. Division of τ' by τ_A then yielded the photographic contrast-transfer function.

It was found that for all three films the contrast-transfer function was substantially the same for contrasts up to 0.5 density units within the density range 0.5 to 1.5. The various contrast-transfer functions measured within this range are shown in Fig. 2.

Measurements of the noise were made by using the microdensitometer to scan across uniformly exposed regions of film. The effective scanning slit was again $0.5 \times 150\mu$, and a one-dimensional trace was obtained on the recorder chart of the density fluctuations due to granularity, averaged over this area. By numerical analysis of the trace, and use of the Ilford Limited Leo II computer, the noise power spectrum was calculated. 1,200 readings taken from the trace at intervals of 0.5μ gave an over-all uncertainty of around 10% in the noise power in the range 0 to 200 lines/mm. *

* For a detailed discussion of the dependence of the accuracy of this method on the number of readings and the interval, see Zweig.[20]

20. H. J. Zweig, *J. Opt. Soc. Am.*, **46:** 805 (1956).

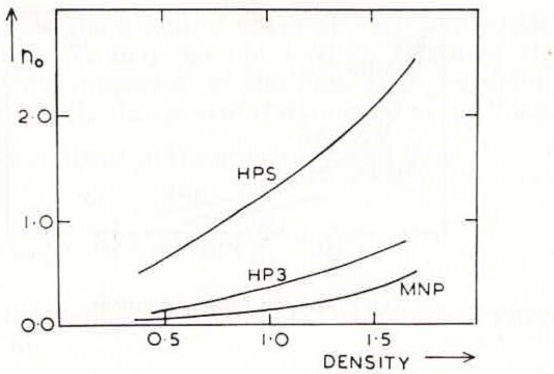

Fig. 3 The dependence of the noise power on the mean density level. The units of noise power are $\mu^2 \times$ density^2.

Again the effects of the microdensitometer system had to be allowed for, this time by use of the relation

$$N'(\omega) = \tau^2{}_A N(\omega)$$

where $N'(\omega)$ is the measured noise power spectrum and $N(\omega)$ is that due to the film alone. The calculations showed that $N'(\omega)$ followed the shape of $\tau^2{}_A$ (to within the experimental uncertainty) and that this shape was the same for each film and was independent of the mean density level in the density range 0.2 to 1.5. This indicates that the noise power spectrum of each film is substantially flat, i.e. n_0 is independent of spatial frequency. The values of n_0 for each film, and their variation with mean density level, are shown in Fig. 3.

Information Capacity

From Eqs. (4) and (5) (see *Formulation*) it is seen that the signal-to-noise ratio is given by

$$S(\omega) = \frac{0.189\,\gamma^2\,\tau^2\,(\omega)\,p_0}{n_0(\omega)}$$

For low spatial frequencies this reduces to

$$S_0 = \frac{0.189\,\gamma^2 p_0}{n_0}$$

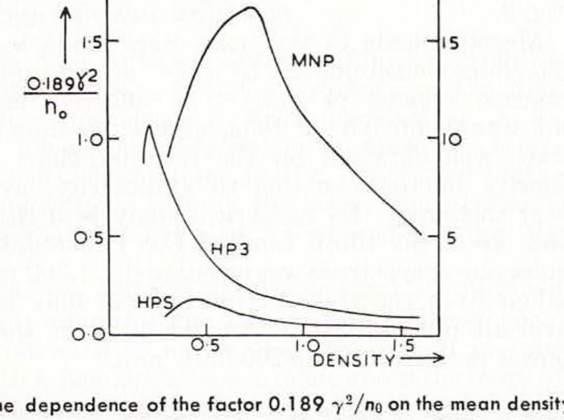

Fig. 4 The dependence of the factor 0.189 γ^2/n_0 on the mean density level. The units are μ^{-2}; the scale on the left is for HPS and HP3, while that on the right is for Micro-Neg Pan.

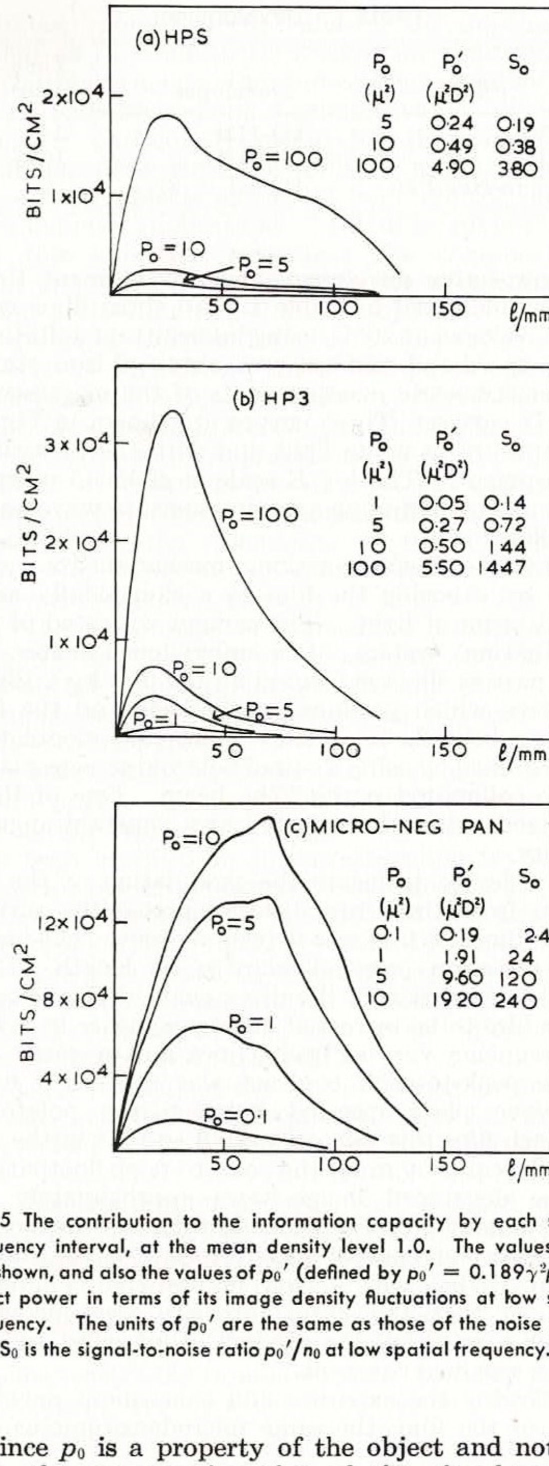

Fig. 5 The contribution to the information capacity by each spatial frequency interval, at the mean density level 1.0. The values of p_0 are shown, and also the values of p_0' (defined by $p_0' = 0.189\gamma^2 p_0$), the object power in terms of its image density fluctuations at low spatial frequency. The units of p_0' are the same as those of the noise power n_0. S_0 is the signal-to-noise ratio p_0'/n_0 at low spatial frequency.

Since p_0 is a property of the object and not the film, the comparative size of the signal-to-noise ratio will be governed by the factor $\dfrac{0.189\ \gamma^2}{n_0}$. This factor is closely related to the quantity defined as the equivalent number of photons by Fellgett,[21] who investigated its dependence on the mean density level for various film types. From Fig. 4 it is seen that in the present case it is as low as $0.03/\mu^2$ for HPS at a mean density level of 1.50 and as high as $15.6/\mu^2$ for Micro-Neg Pan at a mean density level of 0.75. It is highest for Micro-Neg Pan at

21. P. B. Fellgett, *Monthly Notices Roy. Astron. Soc.*, **118**: 224 (1958).

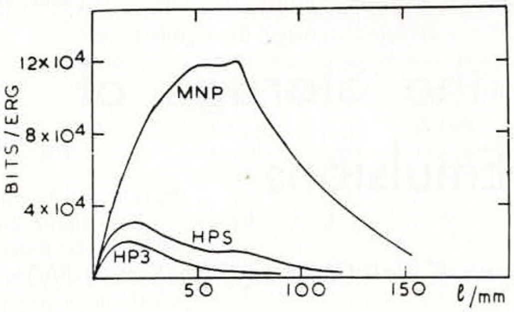

Fig. 6 The contribution to the information capacity of the three films by each spatial frequency interval. In each case this has been evaluated for $p_0 = 5\ \mu^2$ at the mean density level 1.0.

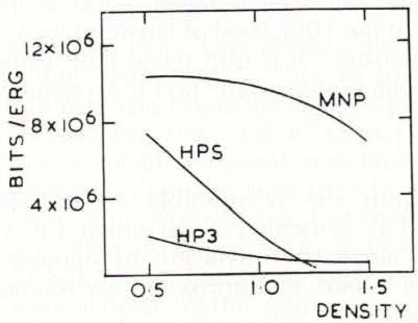

Fig. 7 The variation of the total information capacity of each film with mean density level for $p_0 = 5\ \mu^2$, obtained by use of Eq. (5).

all density levels, since this film has the lowest noise and highest contrast.

Due to this higher signal-to-noise ratio, Micro-Neg Pan again emerges best when informational assessments are made of image quality by use of Eq. (4).

Figs. 5a, b, and c show the results for each of the films in terms of the distribution of the information capacity (in bits/cm²) over the spatial frequency plane. For each film the results (at a mean density level of 1.0) are shown for various values of p_0. Also given are the respective values of p_0', defined by $p_0' = 0.189\ \gamma^2\ p_0$; p_0 is the size of the power spectrum of the object intensity fluctuations, and p_0' is that of the resulting density fluctuations in the developed image, having the same units ($\mu^2 \times$ density²) as the noise power. For each film and each value of p_0 the signal-to-noise ratio at low spatial frequencies $\left(S_0 = \frac{p_0'}{n_0}\right)$ is given. The total information capacity (for each particular value of p_0) is yielded by integration over the spatial frequency plane, and hence corresponds to the area under the curve. As expected, by this assessment the films are rated in order of increasing granularity, with HPS emerging last.

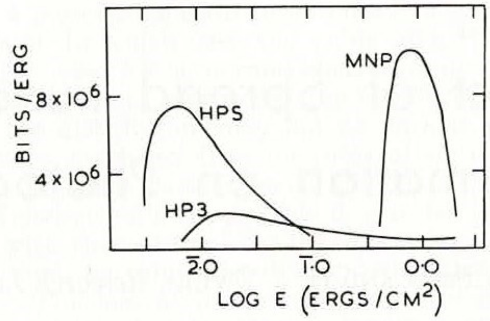

Fig. 8 The variation of the total information capacity of each film with mean exposure level for $p_0 = 5\ \mu^2$, obtained by use of Eq. (5).

When information assessments of the over-all photographic efficiency are made by use of Eq. (5), so introducing the speed, the films are rated in a different order. Fig. 6 shows the distribution of the information capacity (in bits/erg) over the spatial frequency plane. Again the total information capacity of each film (for $p_0 = 5\ \mu^2$ at a mean density of 1.0) is given by the area under the respective curve. By this assessment the difference between the three films is much smaller, and although Micro-Neg Pan is still rated best, HPS is now ahead of HP3.

The variation with density level of the total information capacity (obtained by carrying out the integration over the spatial frequency plane) is shown for each film in Fig. 7. Here the results are again given in bits/erg for $p_0 = 5\ \mu^2$. Fig. 8 shows the variation of this same quantity with the mean exposure level, which indicates the possible range of application of each film and its over-all efficiency within this range. Clearly, of the three, HPS is the obvious choice at low exposure levels, and Micro-Neg Pan, that at high levels. HP3 compensates for its lower efficiency by its high exposure latitude between these two levels.

With the methods of measurement described under *Necessary Measurements*, the possible inaccuracy in the results shown in Fig. 8 is estimated at around 30%.

Acknowledgment

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